A Step by Step Analytical Solution to the Single Diode Model of a Solar Cell

H. A. Koffi[1], E. A. Armah[1], K. Ampomah-Benefo[1], D. Dodoo-Arhin[2]
[1]Dept. of Physics, School of Physical and Mathematical Sc., University of Ghana, LG 63, Legon
[2]Dept. of Material Sciences, School of Engineering, University of Ghana, LG 77, Legon
Corresponding Author: Hubert Azoda Koffi; hakoffi@ug.edu.gh
Received: 01-11-2022, Received in Revised form: 21-12-2022, Accepted: 21-12-2022, Published: 31-12-2022

Abstract

Making use of previous results where the series resistance, R_s, and the light-generated current, I_L, of a solar cell are determined through the knowledge of the open-circuit voltage, V_{oc}, the short circuit current, I_{sc}, the voltage and current at the maximum power point, V_{mp} and I_{mp}. A simple and analytical step by step approach has been developed to determine the shunt resistance, R_{sh}, the reverse saturation current, I_s and the ideality factor, A, of a solar cell. Making use of these results and with the knowledge of the operating temperature, T, this work demonstrates that a single I-V curve is enough to fully solve the transcendental equation governing the behaviour of a solar cell in the Single-Diode Model.

Keywords: Solar cell, Single-Diode Model, Series resistance, Light-generated current, Shunt resistance, Short circuit current, Reverse Saturation Current.

Introduction

The Single-diode model

Modeling a solar cell is a basic means for extracting the effective operating values of the parameters governing the behaviour of the device. For the determination of these parameters, various models have been developed. The most widely used among existing models is the Six-Parameter model or Single-Diode model. In this model, the equation governing the behaviour of the cell is formulated as a transcendental exponential equation involving six parameters namely the light-generated current (I_L), the reverse saturation current (I_S), the operating temperature, (T), the shunt resistance, (R_{sh}), the series resistance, (R_S), and the ideality factor, A. The most investigated parameter out of the six is the series resistance. This is because the series resistance affects the output power of the solar cell more than any other parameter. However, the shunt resistance which determines the leaking currents along the edges of the solar cell is equally important in the analysis of the performance of a solar cell [1]. Also, in some models such as the Five-Parameter model, the ideality factor, which is a measure of how close the equation describes the ideal diode, is arbitrarily set to unity [2]. Hence, the single-diode model is represented by the equivalent circuit shown in Figure 1. The circuit consists of:

(i) a current generator representing the source of current production when the inherent voltage across the junction is connected to an external load
(ii) a diode expressing the requirement of a threshold energy level for the photons to trigger a significant production and circulation of paired electrons and holes across the junction
(iii) a series resistance representing the metal-semiconductor contact resistance, the ohmic resistance in the metal contacts, and the ohmic resistance in the semiconductor material
(iv) a shunt resistance representing the leaking currents along the edges of the solar cell

From Figure 1, the current produced by the solar cell is equal to that produced by the current source, I_L minus that which flows through the diode, I_D, and the shunt resistor, I_{Sh}. The equation governing the current flowing across the load is hence given by equation 1 [2]:

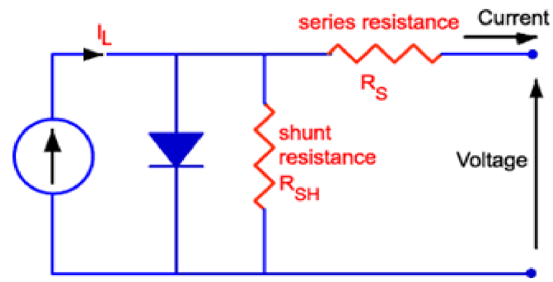

Figure 1: The equivalent circuit of a solar cell

$$I = I_L - I_s\left[e^{\left[\frac{q(V+IR_S)}{Ak_BT}\right]} - 1\right] - \frac{V+IR_S}{R_{sh}} \quad (1)$$

where I is the output current, T the operating temperature, I_s the reverse saturation current, q the unit electrical particle charge, K_B, Boltzmann constant and A the ideality factor. Therefore, the relevant six parameters of the model are T, I_L, I_s, R_s, R_{sh} and A.

To determine one or more of these parameters, several approaches ranging from analytical to computer-based solutions have been developed [2]. In this paper, a simple and analytical approach, shown to underpin an existing computer-based approach in determining the series resistance of a solar cell has been used.

Experimental solutions

One of the earliest approaches to the solution of equation 1 is experiment-based and involves the determination of R_s through the measurement of the output current of the cell which is exposed to light of known intensity [3]. Indeed, it is the case that most studies are concerned with just only one parameter of the photovoltaic phenomenon [4], [5]. Later, several approaches which deal with more than one parameter have been developed and the most common approach involves the use of I-V curves plotted for different irradiances (usually, two values of irradiance) as recommended by the European standard [6]. However, analytical approaches providing more accurate results have been widely used [7].

Analytical approaches

To obtain more accurate results close to the expected values, analytical approaches have been used widely to investigate and determine some or all the parameters of solar cells. Some of them may need mathematical manipulations such as the mean

https://doi.org/10.24949/njes.v15i2.728

value theorem [8] or measured [7]. Others may need existing parameters [9] to determine the investigated parameters. Also, some approaches depend on a model said to be simple to use and known as Rauschenbach's model [10]. In this case, the current generated by the solar device is plotted against the voltage and three characteristic points of this *I-V* curve, namely the short circuit current, (I_s, 0), the open circuit voltage (V_{oc}, 0) and the maximum power point (I_m, V_m) are used to derive the following equation for the current, I [11]:

$$I = I_{SC}\left[1 - C_1 e^{\left[\frac{V}{C_2 V_{oc}} - 1\right]}\right] \quad (2)$$

where

$$C_1 = \left(1 - \frac{I_{mp}}{I_{sc}}\right) e^{\left[\frac{-V_{mp}}{C_2 V_{oc}} 1\right]} \quad (3)$$

and

$$C_2 = \frac{\left[\frac{V_{mp}}{V_{oc}} - 1\right]}{log\left[1 - \frac{I_{mp}}{I_{sc}}\right]} \quad (4)$$

where V_{mp}, V_{oc}, I_{mp}, and I_{sc} carry their usual meanings.

Special functions-based solutions

One form of solution uses Lambert W-function [12], [13]. This function, defined as the reverse function of xe^x {$W[u(x)e^{u(x)}] = u(x)$} and well established in electronics, is used to determine either some or the whole set of the parameters of a solar cell. This approach has been extended to organic solar cells [14], [15].

Another form of this approach is the use of the Special Trans Function Theory (STFT) to determine the series resistance and the ideality factor. [16].

An additional approach to those special-functions-based solutions has been developed by normalizing the current and voltage by dividing observed values of current (or voltage) by the short circuit current (or Open-circuit voltage). One major advantage of this method is that one may obtain the maximum power point without iterative calculations. [17].

Simulations or Digital Forms of solutions

Digital forms or computer-based solutions of the equations governing the output of solar devices are most often used when the behaviour of the devices are simulated either experimentally in a laboratory or with a computer. Some of the simulation software used are AMPS-1D® (Analysis of Microelectronic and Photonic Structures for One Dimension) developed by the Center for Nanotechnology Education and Utilization of Pennsylvania State University.

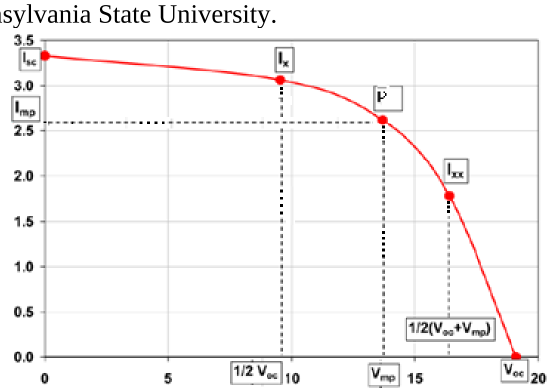

Figure 2: A sample I-V curve illustrating the five operating points provided by the Sandia performance model [18]

These are very general codes for simulating the current-voltage behaviour of solar devices, among others. Some other digital forms are customized to the technology or equipment used to simulate the solar device. Among them are the Sandia National Labs Model [18], [19], and the Agilent Technology method to determine the series resistance and the shunt resistance.

The Sandia National Labs Model predicts the electrical output at short circuit, maximum power, open-circuit, midway between short circuit and maximum power, and midway between maximum power and open-circuit, as illustrated by the points in Figure 2.

In the Agilent Technology method [20], [21], a graphical approach (Figure 3) and a set of equations (equations 5 to 8) are used to determine the necessary parameters, as shown below.

$$V = \frac{\frac{V_{oc} \times \ln\left[2 - \left(\frac{I}{I_{sc}}\right)^N\right]}{\ln(2)} - R_S \times (I - I_{sc})}{1 + \frac{R_S \times I_{sc}}{V_{oc}}} \quad (5)$$

$$R_S = \frac{V_{oc} - V_{mp}}{I_{mp}} \quad (6)$$

$$N = \frac{\ln(2 - 2^a)}{\ln\left(\frac{I_{mp}}{I_{sc}}\right)} \quad (7)$$

$$a = \frac{V_{mp} \times \left(1 + R_S \times \frac{I_{sc}}{V_{oc}}\right) + R_S \times (I_{mp} - I_{sc})}{V_{oc}} \quad (8)$$

where V is the output voltage, V_{mp} and I_{mp} are the voltage and current at the maximum power point respectively, and a and N are inbuilt algorithm-based parameters with N strongly related to the shunt resistance.

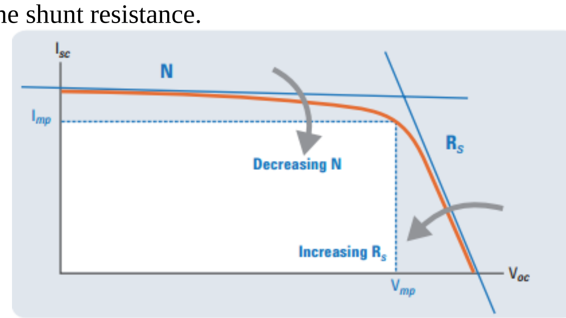

Figure 3: The Agilent Technology method to determine the series and the shunt resistances [20]

Background: A new formulation of The Shockley Five-Parameter model of a solar cell

Previous works [22], [2] have shown that equation 1 can be presented as follows:

$$I\left(1 + \frac{R_s}{R_{sh}}\right) = I_L - I_s\left[e^{\left[\frac{q(V+IR_s)}{AkT}\right]} - 1\right] - \frac{V}{R_{sh}} \quad (13)$$

and, by dividing both sides by $\alpha_K = \left(1 + \frac{R_s}{R_{sh}}\right)$ (14)

$$I = \frac{I_L}{\left(1 + \frac{R_s}{R_{sh}}\right)} - \frac{I_s}{\left(1 + \frac{R_s}{R_{sh}}\right)}\left[e^{\left[\frac{q(V+IR_s)}{AkT}\right]} - 1\right] - \frac{V}{R_s + R_{sh}} \quad (15)$$

hence established that

$$I = \left[I_{sc} - \frac{V}{Rs + R_{sh}}\right] - \left[\frac{I_s}{\left(1 + \frac{R_s}{R_{sh}}\right)}\left(e^{\left[\frac{q(V+IR_s)}{AkT}\right]} - 1\right)\right] \quad (16)$$

Where

$$I_{sc} = \frac{I_L}{1 + \frac{R_s}{R_{sh}}} \quad (17)$$

$$I_{SK} = \frac{I_s}{\left(1 + \frac{R_s}{R_{sh}}\right)} \quad (18)$$

and

$$G_K = \frac{1}{R_K} = \frac{1}{Rs+R_{sh}} \tag{19}$$

is the slope of the linear component guiding the behaviour of the device before the maximum power point.
Hence equation 16 becomes

$$\mathrm{I}=\left[\mathrm{I}_{sc} - \frac{V}{R_K}\right] - \left[I_{SK}\left(e^{\left[\frac{q(V+IRs)}{AkT}\right]} - 1\right)\right] \tag{20}$$

Furthermore, it has been shown [22] that equation 13 is the same as

$$\mathrm{I}=\mathrm{I}_{\mathrm{SC}}\left[1 - e^{\left[\frac{q(V+IRs-Voc)}{AkT}\right]} - e^{\left[\frac{-qVoc}{AkT}\right]}\right] - \frac{V}{Rs+R_{sh}} \tag{21}$$

In line with the above set of equations, previous works have established that:

- the light-generated Current, I_L, can be expressed as a function of the short circuit current, I_{SC}, and the series and shunt resistances, R_S and R_{sh} respectively [23]:

$$I_L = \left(1 + \frac{R_s}{R_{sh}}\right) I_{sc} \tag{22}$$

- the series resistance, R_s, can be expressed as follows [2]:

$$\mathrm{R_S} = \frac{\mathrm{V_{oc}} - \mathrm{V_{mp}}}{\mathrm{I_{mp}}} \tag{23}$$

NB: Since the European Standard [6] imposes that the process of characterizing a solar cell through its I-V curve should be performed at constant temperature and irradiance, the operating temperature is most often known or given. Also, recalling that the relevant parameters are I_L, I_s, T, R_s, R_{sh}, and A, and making use of these two results, this work focuses on the determination of the other remaining parameters in an attempt to obtain a general solution to the six-parameter model or single-diode model of a solar cell.

An analytical step by step solution

The Shunt resistance

As illustrated by Figure 4 [24], it has been established that the shunt resistance affects mostly the output current and subsequently the output power of a solar cell before the maximum power point.

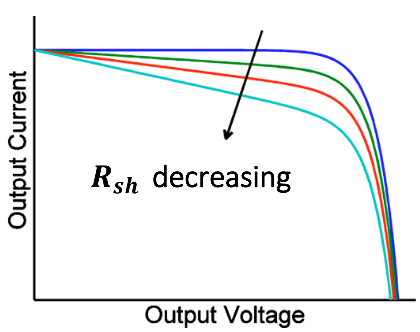

Figure 4: Effect of reducing the shunt resistance [24]

From equation 1, one can derive

$$I_L = I + I_s\left[e^{\left[\frac{q(V+IR_s)}{Ak_BT}\right]} - 1\right] + \frac{V+IR_s}{R_{sh}} \tag{24}$$

Applied at the short circuit current and the maximum power point, respectively, equation 24 becomes

$$I_L = (1 + \frac{R_s}{R_{sh}})I_{sc} + I_s\left[e^{\left[\frac{q(I_{sc}R_s)}{Ak_BT}\right]} - 1\right] \tag{25}$$

and

$$I_L = (1 + \frac{R_s}{R_{sh}})I_{mp} + \frac{V_{mp}}{R_{sh}} + I_s\left[e^{\left[\frac{q(V_{mp}+I_{mp}R_s)}{Ak_BT}\right]} - 1\right] \tag{26}$$

By equating component by component equations 25 and 26, it can be shown that

$$(1 + \frac{R_s}{R_{sh}})I_{sc} = (1 + \frac{R_s}{R_{sh}})I_{mp} + \frac{V_{mp}}{R_{sh}} \tag{27}$$

and multiplying both sides by R_{sh} leads to

$$(R_s + R_{sh})(I_{sc} - I_{mp}) = V_{mp} \tag{28}$$

hence

$$(R_s + R_{sh}) = \frac{V_{mp}}{I_{sc} - I_{mp}} \tag{29}$$

or

$$(R_s + R_{sh}) = \frac{1}{G_K} = R_K = \left|\frac{V_{mp} - 0}{I_{mp} - I_{sc}}\right| \tag{30}$$

Consequently,

$$R_{sh} = R_K - R_s \tag{31}$$

where R_K is the reverse of the slope between the maximum power point and the short circuit current.
Alternatively, the shunt resistance can be determined using equation 21. Indeed, it can be shown that

$$V_K = V + IRs - V_{oc}\begin{cases} < 0 & \text{before the maximum power point} \\ = 0 & \text{at the maximum power point} \\ > 0 & \text{after the maximum power point} \end{cases} \tag{32}$$

so that for $0 \leq V \leq V_{mp}$

$$1 - e^{\left[\frac{q(V+IRs-Voc)}{AkT}\right]} - e^{\left[\frac{-qVoc}{AkT}\right]} \cong 0 \tag{33}$$

Hence

$$\mathrm{I} = \mathrm{I}_{\mathrm{SC}} - \frac{V}{Rs+R_{sh}} = \mathrm{I}_{\mathrm{SC}} - \frac{V}{R_K} \tag{34}$$

The present approach for determining the shunt resistance of a solar cell is simpler as compared to the Agilent Technology method in which N as defined by equations 7 and 8 is said to be strongly related to R_{sh}.
Park and Choi [25] have shown that the maximum possible value of the shunt conductance, G_{sh}, is:

$$0 \leq G_{sh} \leq \frac{I_{sc} - I_{mp}}{V_{mp}} \tag{35}$$

Indeed, and per the results in that work, $G_{sh} = \frac{I_{sc}-I_{mp}}{V_{mp}}$ only and if only $R_s \approx 0$. But that is not always the case. Therefore, it can be said that the present approach gives better accuracy than the Park and Choi results.
Also, existing work [26], defining $U = V + IR_s$ and decomposing before the maximum power point the exponent component of equation 1 in the form of a Taylor series limited to the first degree, i.e.

$$e^{\left[\frac{qU}{AkT}\right]} \approx 1 + \frac{qU}{AkT} \tag{36}$$

has shown that:

$$R_{sh} = \frac{1 + \frac{U_{oc} - U_{mp}}{e^{\left[\frac{qU_{oc}}{Ak_BT}\right]} - e^{\left[\frac{qU_{mp}}{Ak_BT}\right]}} \times \frac{q}{Ak_BT}}{\frac{I_{sc} - I_{mp}}{U_{mp}} - \frac{I_{mp}}{e^{\left[\frac{qU_{oc}}{Ak_BT}\right]} - e^{\left[\frac{qU_{mp}}{Ak_BT}\right]}} \times \frac{q}{Ak_BT}} \tag{37}$$

Then, assuming that expression

$$\frac{1}{e^{\left[\frac{qU_{oc}}{Ak_BT}\right]} - e^{\left[\frac{qU_{mp}}{Ak_BT}\right]}} \times \frac{q}{Ak_BT} \tag{38}$$

is a very small value, that may not always stand, he concludes that

$$R_{sh} = \frac{U_{mp}}{I_{sc}-I_{mp}} \tag{39}$$

The result obtained by the present work agrees with the results of previous approaches. It is also a simpler and improved way of determining R_{sh}.

The ideality factor

As has already been pointed out, the ideality factor, A, is a measure of how closely the basic Shockley equation describes the ideal diode. Therefore, its determination is important for the full application of the Shockley equation. Figure 5 [27] shows that variations in the ideality factor, **A**, affect the output power of the solar cell, mainly after the maximum power point.

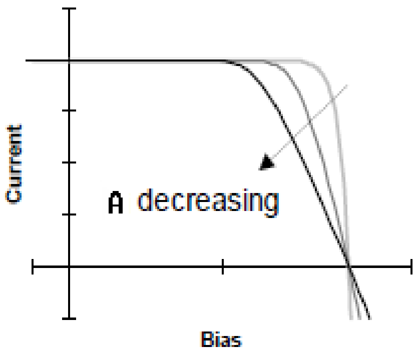

Figure 5: Effect of reducing the ideality factor [27]

The ideality factor, A, affects the efficiency of the solar cell, hence, cells are made "ideal" by reducing the ideality factor to unity [28].

Equation 20 is recalled and rearranged as follows:

$$I = I_{SC} - \left\{I_{SK}\left[e^{\left[\frac{q(V+IRs)}{AkT}\right]} - 1\right] + \frac{V}{R_s+R_{sh}}\right\} \tag{40}$$

It is also recalled that for T = 298 K (25 °C), $1 \leq A \leq 2$ and $V \geq 0.5\,V$, $\frac{qV}{AkT} \geq 20$, hence at higher voltages, the exponential term that contains the ideality factor, A, is very large, so that

$$\frac{V}{R_s+R_{sh}} \ll I_{SK}\left[e^{\left[\frac{q(V+IRs)}{AkT}\right]} - 1\right] \tag{41}$$

and

$$e^{\left[\frac{q(V+IRs)}{AkT}\right]} \gg 1 \tag{42}$$

Thus, equation 20 is reduced to

$$\text{I=Isc- } I_{SK}\left[e^{\left[\frac{q(V+IRs)}{AkT}\right]} - 1\right] \tag{43}$$

So that for any couple of points A and B selected after the maximum power point,

$$I_{SK}e^{\left[\frac{q(V_A+I_AR_s)}{AkT}\right]} = I_{SC} - \quad I_A \tag{44}$$

$$I_{SK}e^{\left[\frac{q(V_B+I_BR_s)}{AkT}\right]} = I_{SC} - \quad I_B \tag{45}$$

$$e^{\left[\frac{q}{AkT}\{(V_A+I_AR_s)-(V_B+I_BR_s)\}\right]} = \frac{I_{SC}-I_A}{I_{SC}-I_B} \tag{46}$$

$$\frac{q}{AkT}[(V_A - V_B) + (I_A - I_B)R_s] = ln\left[\frac{I_{SC}-I_A}{I_{SC}-I_B}\right] \tag{47}$$

hence

$$A = \frac{q}{kT}\left[\frac{(V_A-V_B)+(I_A-I_B)R_s}{ln\left[\frac{I_{SC}-I_A}{I_{SC}-I_B}\right]}\right] \tag{48}$$

An alternative approach is to rewrite equation 43 as follows:

$$e^{\left[\frac{q(V+IRs)}{AkT}\right]} - 1 = \frac{I_{SC}-I}{I_{SR}} \tag{49}$$

that gives

$$\text{V}= \frac{AkT}{q} log\left[1 + \frac{I_{SC}-I}{I_{SR}}\right] - IR_s \tag{50}$$

The above formulation is already used in the literature [29]. Nevertheless, an improvement in this use as suggested by the present work is to select two different points A and B to help eliminate the reduced reverse saturated current I_{SR} from the equation, and, with $I_{SR} \ll Isc$,

$$V_A = \frac{AkT}{q} log\left[1 + \frac{Isc-I}{I_{SR}}\right] - I_AR_S \tag{51}$$

$$V_B = \frac{AkT}{q} log\left[1 + \frac{Isc-I}{I_{SR}}\right] - I_BR_S \tag{52}$$

one can write

$$V_A - V_B = \frac{AkT}{q} log\left[\frac{Isc-I_A}{Isc-I_B}\right] - R_S[\,I_A - I_B] \tag{53}$$

hence

$$A = \frac{q}{kT}\frac{(V_A-V_B)+R_S(I_A-I_B)}{log\left[\frac{I_{SC}-I_A}{I_{SC}-I_B}\right]} \tag{54}$$

which is the same as equation 48 obtained by the present work.

Reverse saturation current

To determine the reverse saturation current, we recall [30] that

$$I_s = I_L e^{\left[-\frac{qV_{OC}}{AkT}\right]} \tag{55}$$

Hence, the reverse saturation current is found to be

$$I_s = \left(1 + \frac{R_s}{R_{sh}}\right) I_{SC}e^{\left[-\frac{qV_{OC}}{AkT}\right]} = \alpha_K\, I_{SC}e^{\left[-\frac{qV_{OC}}{AkT}\right]} \tag{56}$$

Conclusion

In this work, we have shown that knowledge of the four key parameters of an I-V curve, namely the short circuit current, I_{sc}, the voltage and current at the maximum power point, V_{mp} and I_{mp}, and the open circuit voltage, V_{oc}, can be used to determine the five unknown parameters of the single-diode model of a solar cell. To do so, one determines, step by step, in the following order:

- the series resistance expressed against the four key parameters mentioned above,
- the shunt resistance through the determination of the slope of the straight line joining the short circuit current and the maximum power point,
- the light generated current through the short circuit current and the series and shunt resistances,
- the ideality factor through two arbitrary points on the I-V curve, selected after the maximum power point, and
- the reverse saturation current, with the aid of the light generated current and the ideality factor.

Since I-V characteristics analysis assumes operation at constant irradiance and temperature, it follows that one of the parameters, T, in the single-diode model, is known at initio. Therefore, the present work has established a step by step method for solving the single-diode model of a solar cell.

Acknowledgment:

Authors express their gratitude to Emeritus Prof. J. K. A. Amuzu, Dept. of Physics, University of Ghana, Legon, for his insightful guidance.

References

1. A. Goetzberger, J. Knobloch and B. Voss, “Crystalline Silicon Solar cells”, *John Willey & Sons*, Chichester, UK, ISBN 0-471-9744-8, 1998, P. 79.
2. H. A Koffi, A. A. Yankson, A. F. Hughes, K. Ampomah-Benefo and J. K. A Amuzu, 2020, “Determination of Series Resistance of a Solar Cell through its Maximum Power Point” *AJSTID*, https://doi.org/10.1080/20421338.2020.1731073, 2020.
3. H. Rauschenbach, M. Wolf, “Series resistance effects on solar cell measurements”, *Advanced Energy Conversion, **3**, Pergamon Press*, 1963. pp. 455-479.
4. M. Nacar, E. Özer and A. E. Yilmaz, “A six parameter single diode model for photovoltaic modules”, *Journal of Solar Energy Engineering, Transactions of the ASME*, 143(1). 2021. doi: 10.1115/1.4047853.
5. J. Greulich, M. Glatthaar and S. Rein, “Fill factor analysis of solar cell’ current – voltage curves”, *Prog. Photovolt: Res. Appl.*, 18, 2010, pp. 511-515
6. CEI, Comitato Electrotecnico Italiano, “Measurement of photovoltaic current-voltage characteristics” *Photovoltaic devices Part 1, Fascicolo 4350 R, Ed. Prima, April. CEI EN 60904-1*, 82-1, 1998, pp. 12.
7. S. K. Sharma, D. Pavithra, N. Srinivasamurthy and B. L. Agrawal, “Determination of solar cell parameters: an analytical approach”; *Journal of Physics D: Applied physics, 26*, 1993, pp. 1130-1133
8. C. Rodriguez and G. Amaratunga, “Analytic Solution to the Photovoltaic Maximum Power Point Problem”, *IEEE Transactions on circuits and Systems*, 2007, 54(9).
9. M. K; El-Adawi and I. A. Al-Nuaim, “A method to determine the solar cell series resistance from a single I-V characteristic curve considering its shunt resistance: new approach”, *Vacuum* 64, 2002, pp. 33-36.
10. M. Sadok and A. Mehdaoui, “Outdoor testing photovoltaic arrays in Saharan region”, *Renewable Energy 33*, 2008, pp. 2516-2524.
11. F. S. Lasnier, “Photovoltaic system sizing and performance by the comparison of demand and expected radiations”, *International Journal of Solar Energy* 9, 1990, pp. 65-76.
12. E. W. Weirstein, “Lambert W Function”, *MathWorld*, http://www.mathworld. wolfram.com/Lambert.W-Function.html, 2022.
13. M. Rawa, Y. Al-Turki, H. Sindi, M. Calasan, Z. M. Ali and S. H. E. Abdel Aleem, “Current-voltage curves of planar heterojunction perovskite solar cells – Novel expressions based on Lambert W function and Special Trans Function Theory”, *Journal of Advanced Research*. doi: 10.1016/j.jare.2022.03.017, 2022.
14. A. Jain and A. Kapoor, “A new approach to study organic solar cell using Lambert W-function”, *Solar Energy Materials & Solar Cells*, 86, 2005, PP. 197-205.
15. S. Sharma, P. Shokeen, B. Saini, S. S. Chetna, J. Kashyap, R. Guliani, S. Sharma, U. M. Khanna, A. Jain and A. Kapoor, “Exact analytical solutions of the parameters of different generation real solar cells using Lambert W−function”, *Invertis Journal of Renewable Energy*, Vol. 4, N°4, 2014, pp. 154-194.
16. N. S. Singh, A. Jain and A. Kapoor “A new method to determine the optimum load of a real solar cell using special trans function theory (STFT)”, *International Journal of Renewable Energy Research*, 3(2), 2013, pp. 378–382.
17. A. K. Das, “An explicit J-V model of a solar cell using equivalent rational function form for simple estimation of maximum power point voltage”, *Solar Energy*. Elsevier Ltd, 98(PC), 2013, pp. 400–403. doi: 10.1016/j.solener.2013.09.023.
18. D. L. King, W. E. Boyson and J. A. Kratochvill, “Photovoltaic array performance model”, SANDIA REPORT SAND2004-3535, Albuquerque, NM: Sandia National Laboratories, Dec. 2004
19. J. S. Stein, J. Sutterlueti, S. Ransome, C. W. Hansen and B. H. King, “Outdoor PV Performance Evaluation of Three Different Models: Single-Diode, SAPM and Loss Factor Model”, *28th European Photovoltaic Solar Energy Conference and Exhibition*, 2013, pp. 2865–2871.
20. Agilent Technology, “Testing terrestrial solar-powered inverters using solar array simulation techniques”, www.agilenttechnology.com/application_note1.pdf, 2022.
21. Agilent Technology “Generating I-V curves with the Agilent E4360A Solar array simulator using the parameters V_{oc}, I_{sc}, N, and R_s”, www.agilenttechnology.com/application_note5.pdf, 2022.
22. H. A. Koffi, “Influence of climatic parameters on the performance of silicon solar panels in southern Ghana”, Ph.D. Thesis, Department of Physics, University of Ghana, Ghana, 2013.
23. H. A. Koffi, A. Kuditcher,V. C. K. Kakane, E. A Armah, A. A. Yankson and J.K.A. Amuzu, “The Shockley five-parameter model of a solar cell: A short note”, *AJSTID*, Vol. 7, Iss N° 6, 2015, pp 481-484, https://doi.org/10.1080/20421338.2015.1038007.
24. S. Dongaonkar, J. D. Servaites, G. M. Ford, S. Loser, J. Moore, R. M. Gelfand, H. Mohseni, H. W. Hillhouse, R. Agrawal, M. A. Ratner, T. J. Marks, M. S. Lundstrom, and M. A. Alam, “Universality of non-Ohmic shunt leakage in thin-film solar cells”, *Journal of Applied Physics* N°108, 124509, 2010, doi:10.1063/1.3518509.
25. J.-Y. Park, S.-J. Choi, “A New PSIM Model for PV Panels Employing Datasheet-based Parameter Tuning”, *The Transactions of the Korean Institute of Power Electronics*, Vol. 20, No. 6, 2015, http://dx.doi.org/10.6113/TKPE.2015.20.6.498
26. A. I. Kanareykin, “Determination of the shunt resistance of a solar cell from its light volt-ampere characteristic”, *IOP Conference series: Earth and Environmental Science*, https://iopscience.iop.org/article/10.1088/1755-1315/979/1/012185/pdf, 2022
27. Worldscibooks, “The Physics of Solar Cells”, *World Sciences Books*, P.14, www.Worldscibooks-comp.com/pdf, 2023.
28. C. Shu-Park, “Circuits”, *The electrical engineering handbook*, 3rd Ed., Dorf R. C., CRC Press LLC, eBook ISBN: 978-1-4200-4976-3, 2000, p. 264.
29. L. Partain, “Solar cell device physics”, *Solar cells and their application*, 2nd Ed., J. Wiley & Sons, Inc., 2010, p. 74
30. E. L. Meyer, “Extraction of Saturation Current and Ideality Factor from Measuring Voc and Isc of Photovoltaic Modules”, *International Journal of Photoenergy*, vol. 2017, Article ID 8479487, 2017, pp.1-9, https://doi.org/10.1155/2017/8479487.