Two-parameter mechanistic model for the fatigue crack growth of metals

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Abstract

In this paper, a two-parameter mechanistic model for the fatigue crack growth has been developed. Fatigue failure is the major causes of mechanical structural failure. The fatigue failure progress in three stages crack initiation, crack growth and final failure. The fatigue crack growth has been modelled by different approaches, however these approaches are generally empirical. In this paper, a mechanistic fatigue crack growth model is proposed. The striation and its relation to the cyclic load is used for the model development. Scanning electronic microscope results are used to establish relation between striation and crack growth. The developed model is two-parameters. The model has been implemented and validated using experimental data from the literature. The model prediction is satisfactory in region II of the crack growth curve. However, in region I and region III the model deviates from experimental data. It is suggested to incorporate interaction of monotonic and cyclic loading in the mechanistic modelling for the fatigue growth.

Key words: Mechanistic, Fatigue, Striation, SEM, two-parameter.

Introduction

Fatigue failure occurs due cyclic loading. Majority of mechanical structures failure occurs as a result of fatigue phenomenon [1]. In order to design any mechanical structure fatigue of metal play very important role suggested by almost researchers [2]. To increase the reliability and fatigue life of any mechanical structure, investigator have broadly investigated fatigue crack growth in metals under cyclic and monotonic loading in the previous few decades. Fatigue crack growth generally occurs in various opening modes like mode I, mode II and mode III. FCG behaviour are affected by all modes of load cycle comprising monotonic and cyclic loading [3]. Generally fatigue failure occurs in a mechanical structure in three phases crack initiation, crack growth and final failure [4, 5] is illustrated in fig. 1.

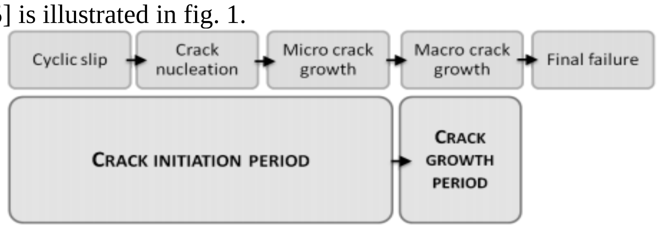

Fig. 1. Various stages of FCG [6]

Crack initiation is the early fatigue damage occurs at the point of maximum stress concentration factor. This phase is controlled by stress concentration factor [7]. Then crack initiation is followed by crack growth. Crack growth is the macroscopic phase of fatigue life occurs in region II about 70% of fatigue life of any materials has been passed in this region reported by various researchers [8, 9]. Various models [10-13] have been developed for the fatigue crack growth prediction. These various models cover to investigate effects of stress ratios, materials and environment on the fatigue crack growth phenomenon.

The existing approaches are generally developed empirically. The physical mechanism of the crack growth has not been well defined and incorporated into the crack growth models. In essence, the crack growth takes place due to plastic flow of the material promoting crack closure. The crack growth behaviour can be explained by the plastic deformation and disbanding of the crack tip. The present knowledge of the fracture surface under fatigue and huge experimental data can be used for the development of a mechanistic fatigue growth model. The mechanistic model will end the hurdles of stress ratio effect and variable amplitude effects on the fatigue crack growth. The aim of this paper is to propose two-parameter mechanistic fatigue crack growth model for metals and its implementation and validation by using fracture mechanics approach.

The next section described the mechanism of striation formation. Section 3 described the effect of stress ratio on fatigue striation. Section 4 presented stress ratio effects on FCG rates. Section 5 discussed Methodology. Section 6 incorporate model implementation and validation. Section 7 present results and discussion. In section 8 the conclusions and recommendations are presented.

Mechanism of striation formation

The most important fractographic features of the fatigue fracture is striation [14]. The mechanism of striation formation has been explained by many models. The oldest and popular model about striation was proposed by Laird's Model [15, 16]. Milella et al [17] described the formation of striation that it is the microscopic feature left on the fracture surface which identify the FCG. Striation is generated at the tip of crack due to micro-plasticity. Plastic blunting during loading and re-sharping during unloading lead the formation of striation as shown in fig. 2.

Some authors proposed different models that the relation between striation spacing is equal to macroscopic crack growth rate (v = s) but this equivalency is valid only for a limited range of ΔK [18-20]. There are some cases for which (v = s) but generally striation spacing show deviation from macroscopic crack growth rate [9, 21, 22].

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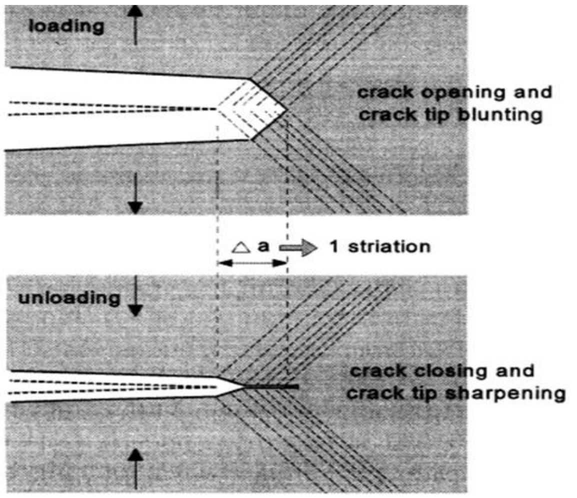

Fig. 2. Crack tip blunting and re-sharping mechanism [17]

Stress ratio and ΔK effect on fatigue striation

Benachour et al [23] investigated that ΔK and mean stress or stress ratio both have the similar effects on the striation spacing i.e. Striation spacing increases with both ΔK and mean stress. At constant amplitude of loading during propagation for different load ratio the striations were observed. It can be observed for the crack growth of same length after the fracture surface of final failure that the load ratio increases fatigue striation spacing as shown in fig. 3.

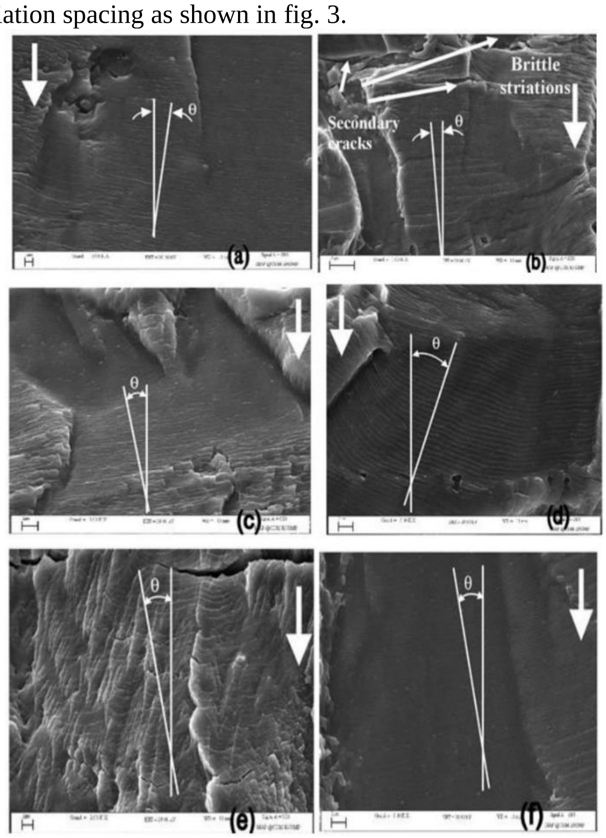

Fig. 3. Fatigue striation spacing at different load ratio [23]

The graph in fig. 4 show us that when the stress ratio effect increases at the same time the fatigue striation spacing also increases for the given value of ΔK. Also, graph shows us stress ratio effect on fatigue striation.

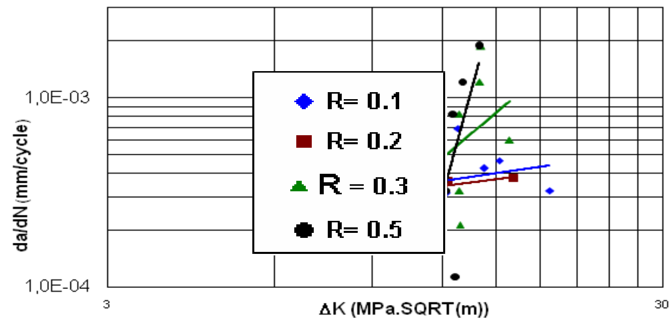

Fig. 4. Striation spacing at different load ratio [23]

Table 1 Striation spacing at different stress ratios [23]

Sr No	Stress Ratio	Crack size	Striation spacing $(\frac{da}{dN})$
A	0.1	5.95 mm	4.28×10-4mm/cycle
B	0.1	7.24 mm	3.23×10-3mm/cycle
C	0.2	5.89 mm	4.0×10-4 mm/cycle
D	0.2	7.00 mm	5.6×10-3 mm/cycle
E	0.3	5.71 mm	5.83×10-4mm/cycle
F	0.3	7.67 mm	3.33×10-4mm/cycle

Stress ratio and ΔK Effects on FCG rates

Furukawa [24] has developed a new approach for the determination of service load effects from fatigue striation spacing during the investigation of fatigue crack growth of 2017 T4 alloy of Aluminium. The relationship between fatigue striation and stress intensity factor range show that crack growth rates is only not influence by ΔK but influence also by load ratio. E.U. Lee et al [25] show the variation of FCG rates versus ΔK in fig. 5 at different load ratios in different environments. While increasing the stress ratio will also increase the fatigue crack growth rate.

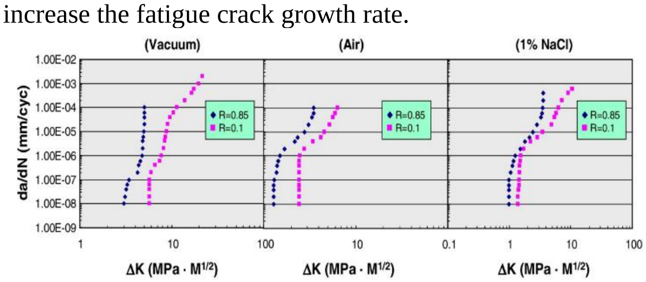

Fig. 5. FCG rates versus ΔK [25].

Methodology

From the above section, we can conclude that FCG rate are the function of two parameters ΔK and stress ratio or mean stress. Various models were developed to accounts the effect of stress ratios and ΔK for the crack growth. The famous model proposed by Forman et al [26] incorporated these two parameters in the model but didn't describe the physical mechanism because the model was empirical based. Benachour et al [23] predict that by increases stress ratio will increased the plastic zone size for the given value of ΔK. If we keep ΔK constant and increase the stress ratio will also increase the plastic zone. Magnus Hörnqvist et al [27] describe that if we keep ΔK constant the fatigue crack growth show variation with corresponding stress ratio.

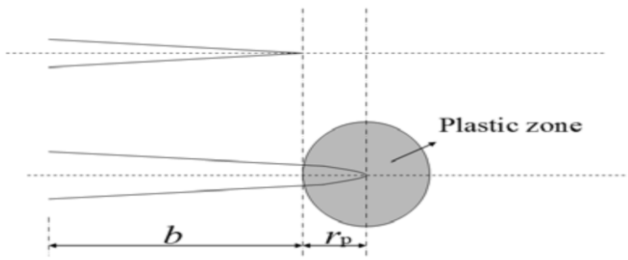

Fig. 6. Plastic zone size formation mechanism [11]

In the above diagram b represent the depth of initial crack and r_p represent plastic zone radius at the tip of crack. The above diagram show us that bluntness will be the function of local material plasticity [11].

If the stress ratio increases for the given value of ΔK thus the plastic zone also increases as show schematically in the following way.

In the following case, we assume that $R_2 > R_1$

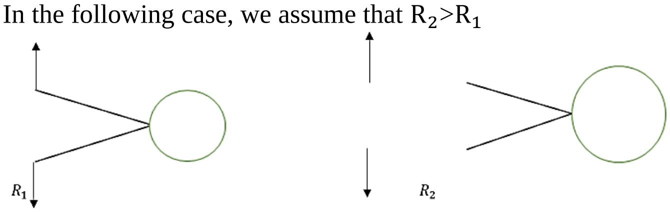

Fig. 7. Schematically plastic zone sizes at R_1 and R_2

It means as the effect of stress ratio increases as a result the plastic zone size will increase thus the striation spacing will increases.

As FCG rates is the function of two parameters according to the above discussion so we can write it in the following form:

$$\frac{da}{dN} = f(\Delta K, \; K_{mean}) \quad (1)$$

Where $\Delta K = K_{max} - K_{min}$, and

$$K_{mean} = \frac{K_{min} + K_{max}}{2}$$

By using the principle of superposition [28] rewrite equation (1) in the following way:

$$\frac{da}{dN} = A(\Delta K)^m + B(K_{mean})^n \quad (2)$$

A, B, m and n are equation parameters determined experimentally

Fatigue phenomenon can be described by any of the following five parameters.

K_{max}, K_{min}, K_{mean}, ΔK and R

The above model given by equation (2) contains two parameters so this suggest that fatigue crack growth is also the function of two parameters. For the fatigue spectrum, at least two parameters are needed. This implies that crack growth rate should be also described by any of two parameters. Fatigue crack growth rate is affected by both types of loading cyclic as well as monotonic loading. The effect of monotonic loading is known as stress ratio effect.

Based upon fractographical analysis, a two-parameter model has been developed in which the contribution of cyclic loading and monotonic loading superimposed rather than multiplied. The above two-parameters model will be validated in the next section with authentic experimental data.

Model implementation and validation

In this section, the model developed in the previous section has been implemented and validated by using open source literature empirical data of fatigue crack growth. Two case studies are briefly discussed below.

Case study I:

The first data set was taken from the work of Chang et al [29]. The material used in their study was 300M steel. The material was tested at various stress ratios. At stress ratios R=0.3, R=0.5 and R=0.7 data of fatigue crack growth is used for the implementation of the current model:

$$\frac{da}{dN} = A(\Delta K)^m + B(K_{mean})^n \quad (3)$$

The test data at stress ratio R=0.05, is used for the validation of above model. The experimental data for the stress ratios 0.7,0.5 and 0.3 is plotted in 3D plot as shown in the fig. 8.

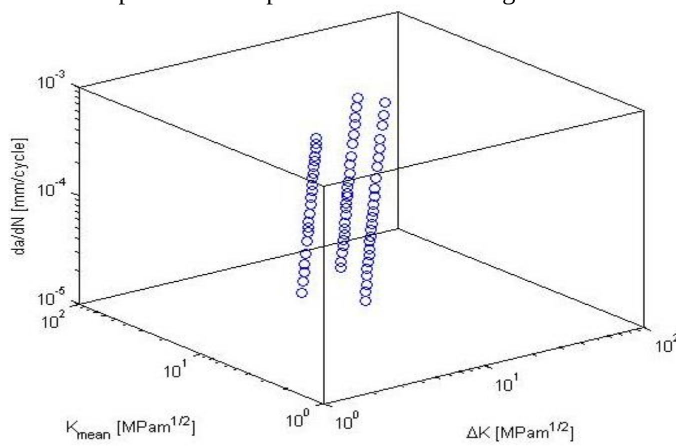

Fig. 8. Crack growth data $\frac{da}{dN}$ verses ΔK and K_{mean} [29]

The model parameters were attained by using surface fitting tool of MATLAB. After several trials, the values of the model parameters were chosen for the highest R^2, equal to 0.9606.

The model parameters were as under.

$A = 5.437e^{-08}$, m=2.22

$B = 1.7408e^{-08}$, n=2

The surface fitting to the experimental data is shown in 3D plot of $\frac{da}{dN}$ verses ΔK and K_{mean} in figure 9.

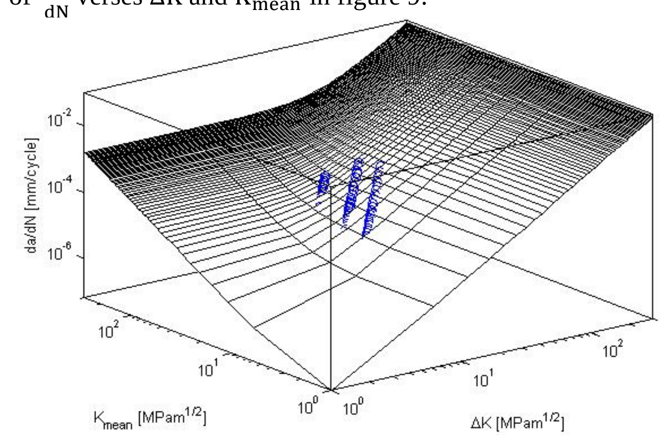

Fig. 9. Surface fitting to the data in figure 8

The final model is given by equation 4.

$$\frac{da}{dN} = 5.437e^{-08}\Delta K^{2.22} + 1.7408e^{-08}K_{mean}{}^{2} \quad (4)$$

For the current model validation, the equation 9 is transformed to single variable in term of ΔK as given in equation 5:

$$\frac{da}{dN} = 5.437e^{-08}\Delta K^{2.22} + 6.61e^{-08}\Delta K^{2} \quad (5)$$

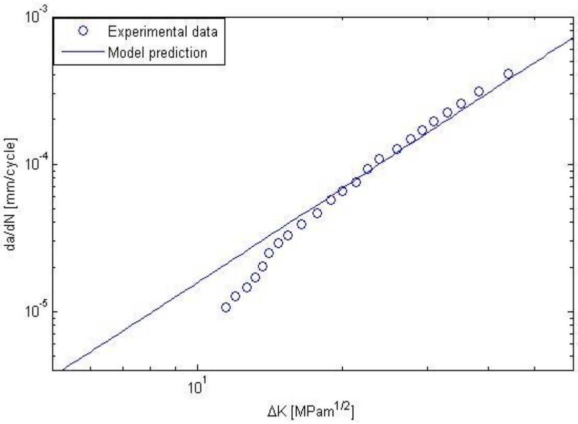

Fig 10. Model validation of case study I.

Fig. 10 shows 2D plot of $\frac{da}{dN}$ versus ΔK to analyse the performance of model validation. The figure shows an exceptional relationship between empirical data and model prediction in the linear region. However, at lower ΔK the deviation of the experimental data from model prediction is seen. The model prediction is slightly higher than the experimental data as show in fig.10.

Case study II

The data set used in this case was taken from the work of Dubey [30]. The material used in their experiment was Ti–6Al–4V. During performing his experiment on the given material, they used various stress ratios. At stress ratios R=0.25, R=0.5 and R=0.8 data of fatigue crack growth is used for the implementation of the following model:

$$\frac{da}{dN} = A(\Delta K)^{m} + B(K_{mean})^{n} \quad (6)$$

But the remaining test data at stress ratio R=0.02 is used for the validation of above model. The experimental data at stress ratios R=0.25, R=0.5 and R=0.8 is plotted in 3D plot form as shown in the fig. 11.

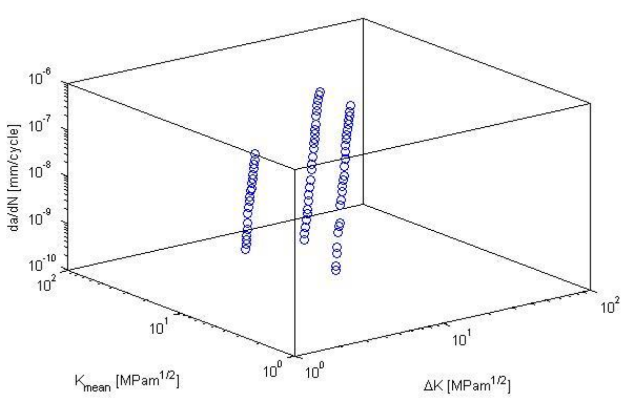

Fig. 11. Crack growth data $\frac{da}{dN}$ verses ΔK and K_{mean} [30]

The model parameters were obtained with the help of surface fitting tool of MATLAB. By doing several trials in surface fitting tool values of the model parameters were chosen at the highest value of R^2, which was equal to 0.7263.

The model parameters were as under.

$A= 3.221e^{-12}$, m=2.4

$B=8.963e^{-12}$, n=2.66

According to the above model parameters the following shape of the model is made:

$$\frac{da}{dN} = 3.221e^{-12}\Delta K^{2.4} + 8.963e^{-12}K_{mean}{}^{2.66} \quad (7)$$

For the current case, the surface fitting to the experimental data as shown in the 3D plot form of $\frac{da}{dN}$ verses ΔK and K_{mean} in fig.12.

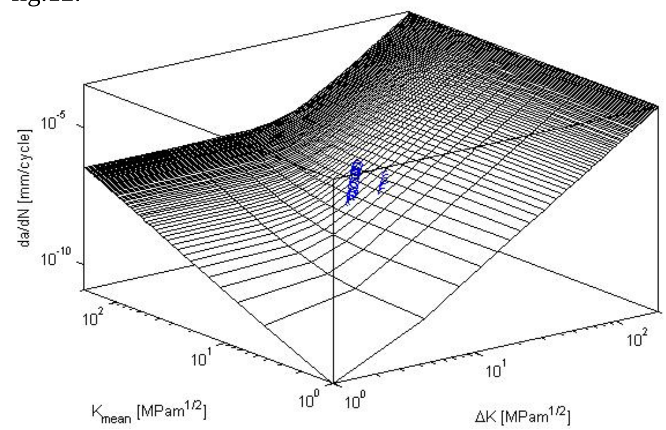

Fig. 12. Surface fitting to the data in figure 11

For the validation of the current model equation 7 is transformed into single variable form in term of ΔK as given by equation 8:

$$\frac{da}{dN} = 3.221e^{-12}\Delta K^{2.4} + 4.666e^{-11}\Delta K^{2.66} \quad (8)$$

To check the performance of the current model validation from the fig. 13 in 2D form of $\frac{da}{dN}$ verses ΔK. The fig. 13 shows a weak relationship between empirical data and model prediction. At lower value of ΔK empirical data is lower than model prediction. The sensitivity of the model prediction is significantly lower than the experimental data as show in fig. 13.

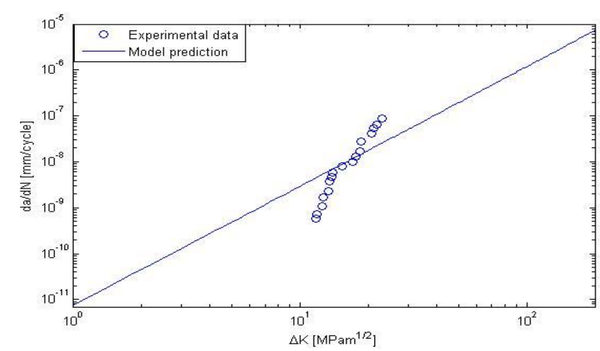

Fig. 13. Model validation of case study II.

Discussion

In this section, modelling of Fatigue crack growth has been discussed in contest with major parameters that play role in its characterization with reference to the current research.

Regions of Fatigue Crack Growth

According to the range of applied stress intensity factor ΔK, FCG curve has sigmoidal shape as shown in fig. 14.

FCG curve can be divided in three regions according to the slope variation in this curve. These regions are often termed as region I, region II and region III. These regions basically show the sensitivity of Fatigue crack growth rate to SIF range.

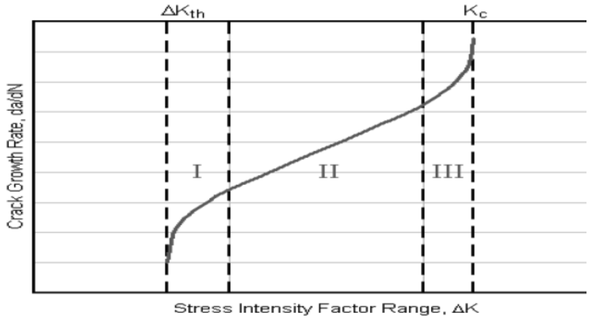

Fig. 14. FCG rates verses ΔK [31]

Region I

In region I, growth rate is more sensitive to the ΔK. The slope of curve is sharp which implies that small variation in ΔK bring large variation in the FCG. In this region, the effect of stress ratio has been reported [23]. This effect is apply to the plasticity induced crack closure [32]. This implies that the fracture surfaces developed in this region fatigue growth should containing significant plastic flow. The striation marks are not continuously observed in this region. The modelling should be formulated that the effect of this plasticity or roughness induced fracture surface is reflected. It may be also assumed that the absence of striation marks revealed that striation is not created in one cycle and for these reason plastic flow of material is responsible for the striation marks which are minute and cannot be detected. The model for FCG will be not the simple superimposition of monotonic and cyclic contribution for crack driving in this region.

Region II

This region is often termed as Paris region. The crack growth curve shape is less steep in this region and it is almost straight line on log-log, $\frac{da}{dN}$ vs ΔK plot. This region spans on a wider range of ΔK as shown in fig14. The model developed in this research work satisfactorily forecast crack growth in this region [31]. This implying that one cycle create one striation mark as show in fig. 14. The stress ratio effect is adjusted into the two-parameter superposition model by default and as further correction is required for this effect. However, the case may vary from one material to another material due to composition difference. It is suggested that mechanistic model is developed in such a way that in region II, it is only the function of monotonic and cyclic load contribution superposition.

Region III

In region III the fatigue crack growth curve slope is sharp implying that it is more sensitive to the ΔK variation. In this region ΔK value approach K_c and non- stable growth is observed. The striation marks are generally not reported in literature for this region growth. The spacing and geometry of fractographic features is non-uniform in this region [14]. It is suitable to characterize crack growth in this region using static fracture criteria rather that fatigue fracture for which static fracture toughness can be used as prime parameter.

Fractographic features of the crack growth and mechanistic model

The fractographic features are important in the modelling of the fatigue growth. Literature studies are present where fractographic features are used for the development of mechanistic model. The important fatigue fracture surface feature is striation marks, that has been extensively used for the development and understanding of the FCG model and fracture process respectively.

However, striation marks are not always observed for the whole SIF range and also for not all materials. Specifically, brittle materials don't exhibit striation due to absence of microplasticity during fracture. It is also still debatable whether striation marks are formed in single cycle or in multiple cycles since the famous models for striation formation are for one cycle [14, 33]. This implies that more fracture features should be involved in the developing of mechanistic model for FCG. A good example of using extra fracture surface features is explored by the Khan's work [34] for composite delamination growth. Though their works is inherently for composites, but the concept can be still extended to metal FCG. Moreover, the model of striation formation should be revised to realistically explain the phenomenon of fatigue fracture.

Similitude parameters for crack growth characterization

The fatigue spectrum is fully described by two loading parameters out of five parameters σ_{max}, σ_{min}, σ_{mean}, Δσ and R as shown in fig. 15.

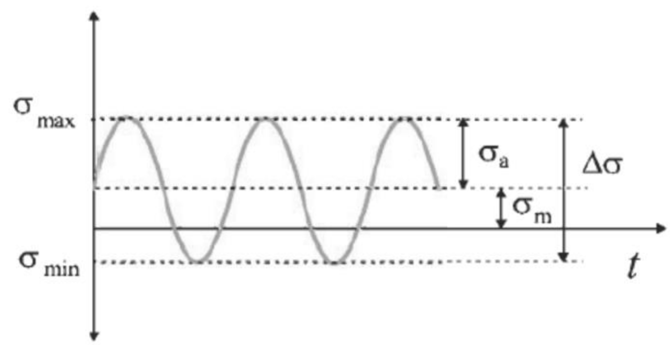

Fig. 15. Loading history of fatigue [35]

The corresponding similitude parameters for FCG are K_{max}, K_{min}, K_{mean}, ΔK and R. In case of strain energy release rate (SERR), the parameters may be G_{max}, G_{min}, G_{mean}, ΔG and R. Analyst are widely divided for the use of a common parameters of FCG. The difference in approaches emanates from the foundation work of Paris equation. Whenever researchers observe that Paris equation was not able to predict FCG for different stress ratio and also different FCG regions, they used various above parameters for characterization. A unified parameter may be developed using the basics of thermodynamics energy conservation principles. An example of such work is Alderliesten [36] approach where the fracture energy required for new surface is related to the loading and from there a parameter has been developed. Further work is however needed to validate Alderliesten model.

Conclusion and recommendation

In this research work, a mechanism-based model has been developed using fractographic observation. Striation marks relation has been used to establish relation between crack growth and loading. A two-parameters model is proposed for the FCG. The model has been implemented and validated using experimental data from literature. Striation marks relation with cyclic loading is more obvious in region II of fatigue crack growth curve. In region II superimposition of two-parameter model is valid for some materials. But in threshold region and region III striation marks are often not observed. Single parameter model does not describe physical phenomenon of FCG.

It should be needed to explore further the interaction of monotonic and cyclic loading and also include in the model to predict all the regions. The model of striation formation should be revised because it's doesn't accurate that one cycle creates one striation mark.

As the striation marks are sometime observed and sometime not observed in threshold and region III so further fractographic features should be used for the modeling the characterization of region III static fracture should be used instead of fatigue fracture.

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